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The set r, s, t is to be rejected if $\sigma(r)$ has a prime factor dividing one of the numbers r, s, t , but not their sum, since it must then divide a . All such sets in which the common divisor sum $\sigma(r)$ does not exceed 360 are given in the following list

$2^2 \cdot 11,$	$5 \cdot 13,$	$83,$	$7\sigma(a) = 2^4a;$
$2 \cdot 5 \cdot 7,$	$5 \cdot 23,$	$7 \cdot 17,$	$3^2\sigma(a) = 19a;$
$5 \cdot 31,$	$7 \cdot 23,$	$191,$	$2^6\sigma(a) = 13^2a;$
$7 \cdot 29,$	$11 \cdot 19,$	$239,$	$2^4 \cdot 5\sigma(a) = 7 \cdot 31a;$
$2 \cdot 5 \cdot 13,$	$5 \cdot 41,$	$13 \cdot 17,$	$3^2 \cdot 7\sigma(a) = 139a;$
$5 \cdot 41,$	$13 \cdot 17,$	$251,$	$2^2 \cdot 3^2 \cdot 7\sigma(a) = 677a;$
$2 \cdot 83,$	$5 \cdot 41,$	$13 \cdot 17,$	$3^2 \cdot 7\sigma(a) = 2^2 \cdot 37a;$
$3 \cdot 89,$	$11 \cdot 29,$	$17 \cdot 19,$	$2^3 \cdot 5\sigma(a) = 101a;$
$11 \cdot 29,$	$17 \cdot 19,$	$359,$	$2^3 \cdot 3^2 \cdot 5\sigma(a) = 7 \cdot 11 \cdot 13a;$
$3 \cdot 89,$	$11 \cdot 29,$	$359,$	$2^3\sigma(a) = 3 \cdot 7a.$

For the final equation the solution $a = 2^5 \cdot 3$ obtained at the end of § 8 is now excluded since 3 divides one of the triple. A valid solution may be found tentatively. Let $a = 2^n b$. For certain values of $n < 14$, a would necessarily be divisible by one of the prime factors of the numbers of the triple; for the others, a solution seems improbable. Take $n = 14$. Then $31 \cdot 151\sigma(b) = 2^{11} \cdot 3b$. Let $b = 31 \cdot 151c$. Then $19\sigma(c) = 2^3 \cdot 3c$. Let $c = 19d$. Then $5\sigma(d) = 2 \cdot 3d$, $d = 5$. Hence we have the amicable triple

$$(X) \quad 3 \cdot 89a, \quad 11 \cdot 29a, \quad 359a, \quad a = 2^{14} \cdot 5 \cdot 19 \cdot 31 \cdot 151.$$

MATHEMATICAL LITERATURE FOR HIGH SCHOOL TEACHERS.¹

By G. A. MILLER, University of Illinois.

There is no sharp line of division between the mathematics for the high school teacher and that which is more advanced. All teachers should strive to increase their knowledge of the subjects which they teach. The teacher who does not hunger and thirst for more knowledge cannot hope to inspire his students with the proper zeal and earnestness. The desire for more knowledge should be based, in part, on the fact that we seek more light on the subjects which we teach.

Mathematics Not a Finished Science.—Even such elementary concepts as those of the natural numbers become more replete with meaning if we study the efforts to base these concepts on those of the theory of aggregates. Questions relating directly to the natural numbers have been considered in recent years by some of the foremost mathematicians, especially in view of the fact that the

¹ Address before the mathematical section of the High School Conference held at the University of Illinois in November, 1912. Reported by Dr. W. W. Denton.

contradictions which have appeared in the theory of infinite aggregates raise the question whether the concept of natural numbers should be based on this theory.¹ This reference may suffice to emphasize the fact that even elementary mathematics is not a finished science, but one which is continually being enriched through the developments in the various fields of our subject.

The Teacher Cannot Stand Still.—Another reason why teachers should strive to increase their knowledge is based on the fact that our intellectual standards are being rapidly raised. This applies to the teachers in the universities as well as to those in the high schools. The fact that a teacher knows enough to meet the demands made upon him to-day does not imply that an equal amount of knowledge will suffice ten years from now. It is well known that some of the men who were appointed as professors in the universities twenty years ago could not meet the requirements of an instructorship to-day. Since most high schools are less conservative than the universities as regards the dismissal of those who do not meet the requirements of the time, it would appear that the high school teacher has special reasons to observe the drift of things.

The Rapid Growth of Libraries.—One evidence of our rapid intellectual advance is furnished by the great growth of our libraries and their facilities to serve the people. The Illinois State University Library, for instance, has doubled the number of its volumes within a decade and is continually becoming a stronger factor in the intellectual advance of the state. About a score of cities in our country contain libraries whose annual income exceeds \$100,000, and the city of Chicago has at least three such libraries. While our libraries are not yet as large as some of those in Europe, we have now at least three containing more than a million volumes each, namely, the libraries of Congress, New York and Boston.

The great development of our libraries implies an active intellectual growth of the people as a whole, and with this growth there naturally comes a demand for higher and better standards on the part of our teachers. To attain these standards it is necessary that we grow, and the most useful aid to intellectual growth is literature. The mathematical literature may be divided into two large classes,—periodic and non-periodic. These classes we shall call journals and books respectively.

Early Mathematical Journals.—The journals are continually assuming a more and more important place in our literature; and they vary so much that almost all classes of readers are now provided for. The first American mathematical journal, known as the *Mathematical Correspondent*, was started about a century ago, but it did not survive long. Several other journals had a similar fate. Our oldest extant mathematical journal, known as the *American Journal of Mathematics*, is less than forty years old, having been started in 1878.

Even in Europe mathematical journals are of a comparatively recent origin. Hence we find ourselves near the beginning of mathematical journals, but we

¹ Cf. C. FÄRBER, "Der Zahlbegriff in Lehrbüchern und im Unterricht," *Archiv der Mathematik und Physik*, vol. 20 (1912), p. 139.

have reason to believe that many of those now extant will live forever. It is interesting to consider the effect of these journals on the future development of our science. As a rule each volume marks an advance over its predecessor, and the 142 volumes of the German journal called, *Journal für die reine und angewandte Mathematik*, furnish some evidence with respect to the elevating influence of the journals of high grade.

Journals for Secondary Teachers.—The most elementary American regular periodicals which are largely or entirely devoted to mathematics are *School Science and Mathematics* and *The Mathematics Teacher*. The AMERICAN MATHEMATICAL MONTHLY is of a little more advanced type, but most of its articles and problems should be within the easy reach of the average high school teacher. The English journal called *Mathematical Gazette* is the organ of their Mathematical Association, formerly known as the "Association for the improvement of geometric teaching." This association aims to bring within its consideration all branches of elementary mathematics, and its journal is of about the same grade as the AMERICAN MATHEMATICAL MONTHLY. The *Annals of Mathematics*, now published under the auspices of Princeton University, is of a somewhat higher grade, but it should be read by the more advanced high school teachers.

Each of the other leading mathematical languages, French, German, and Italian, has similar journals; and these journals are playing an important rôle in the mathematical development of the people using these languages. This implies that periodic literature is filling a want of the teachers of the high school grade in all the leading countries of the world. We are naturally led to inquire why journals are so desirable, and what useful objects they accomplish.

The Influence of Journals.—In the first place, journals bring us face to face with many of the modern questions. They facilitate collaboration and represent the dynamic elements in mathematical literature, while books represent more nearly the static. The following instance may serve to illustrate the point in question. About a year and a half ago a mathematical journal called *Revista de la Sociedad Matemática Española* was started at Madrid, and twenty-five Spanish mathematicians attended the Fifth International Congress of Mathematicians during last summer, while the largest attendance of Spanish mathematicians at any of the preceding congresses was five. Journals seem to accompany the mathematical activity of nations, and hence it is reasonable to assume that they exercise a helpful influence.

It is, of course, true that books have many advantages. They are suitable for the more extended and systematic presentation of subjects and for grading the work so as to put it into a form for the beginner. The point which we desire to emphasize is that the modern teacher needs journals as well as books, and that he should not fail to utilize the advantages offered by the best periodic literature. The loyal support rendered to this literature will tend to improve its quality and to increase its influence for good. We should all aim to enhance the dignity of our profession and to show our interest in it by exerting some influence along lines that tend to ennoble it.

Books of Reference.—While it is comparatively easy to suggest the most suitable mathematical journals, it is much more difficult to give equally good advice with respect to books, since the number of such books is so large. All that we shall aim to do in this connection is to call attention to a few books of reference and to a few works of a general mathematical character.

In 1908 the Royal Society of London published the volume on pure mathematics of its *Subject Index* of the scientific papers which appeared during the nineteenth century. This volume includes 38,748 entries, relating to the mathematical papers which appeared in 700 different serials. It seems to me that all students of mathematics should have access to such reference books, since they serve as a reliable guide to the literature on a large number of different mathematical subjects, and they enable one to get a correct general view of the developed parts of our science. Hence they tend to eliminate the waste involved in doing the same thing over and over without utilizing the results attained by others.

The Great Encyclopedias.—The most valuable modern work of reference, especially for the advanced student of mathematics, is the large mathematical encyclopedia which is being published in German and in French. Many parts of this work relate to subjects which should be of interest to high school teachers. Unfortunately, the parts relating to teaching, history and philosophy are to appear at the end and have not yet been completely planned. The extent of this work may be inferred from the fact that 35 volumes of the French edition have already been planned. It is hoped that the first volume of this edition, devoted to arithmetic, can be completed in the near future. The first volume of the German edition was completed several years ago and it is already out of print.

Valentin's Bibliography.—Another great work of reference, which has been in course of preparation for more than 25 years, should interest all students of mathematics. In 1885 G. Valentin of Berlin, Germany, began to prepare a bibliography of mathematical literature, which is to give a list of all the books and articles that appeared up to 1900. He collected about 150,000 titles, and about one-fourth of these are titles of books, counting different editions and translations of the same book as a single book. This is the most extensive list of mathematical publications that has ever been made, and it is expected that a part of it will be ready for publication during the coming year.

Other General Works.—Those who read English only will doubtless find Carr's *Synopsis of Elementary Results in Pure Mathematics* very useful as a work of reference, even if it is a little old. Among the most modern books in the English language dealing with general mathematics of a high school grade, we may mention the following four: "Monographs on topics of modern mathematics relevant to the elementary field," edited by J. W. A. Young; "Lectures on fundamental concepts of algebra and geometry," by J. W. Young; "Introduction to mathematics," by A. N. Whitehead; and "Easy mathematics, chiefly arithmetic," by Sir Oliver Lodge.

The History of Mathematics.—During recent years there has been a rapidly growing interest in the history of mathematics. This has led to a re-examination

of many matters which had been accepted as historical facts. In a fairly large number of cases it turned out that the earlier conclusions had been based upon insufficient evidence. Hence teachers should be very cautious in reference to historical data. In fact, mathematical histories have been getting out of date so rapidly that it is not safe to place much confidence in some of the conclusions of the histories which are several years old. It is however, true that many important central facts of mathematical history rest on sound foundations and one can often see from the nature of the question and from the evidence presented whether any reasonable doubts remain.

In support of the statement that mathematical histories have been getting out of date very rapidly we may note the fact that in the third edition (1907) of volume I of Cantor's *Vorlesungen über Geschichte der Mathematik*, page 616, there occurs the following sentence: "Ist noch unserem Dafürhalten die Erfindung der Nul eine babylonische, die Vertiefung des Begriffes eine indische, so ist das Rechnen mit der Nul schon zu Brahmaguptas Zeit Gegenstand besonderer Vorschriften gewesen." The corresponding sentence in the second edition (1894), page 576, begins as follows: "Ist noch unserem Dafürhalten die Erfindung der Nul eine indische." The substitution in the newer edition of the word "babylonische" for the word "indische" is important.

The most popular history of mathematics in the English language seems to be Ball's *Short Account of the History of Mathematics*, which reached its fifth edition in 1912 and has been translated into Italian and also into French. That this popularity does not imply unusual accuracy may be seen from the review of the fourth edition, published by G. Eneström in the *Bibliotheca Mathematica*, volume 10 (1910), pages 85 to 88. Many interesting points relating to the early developments of mathematics in our own country are found in Cajori's *Teaching and History of Mathematics in the United States*, 1890. Cajori's *History of Elementary Mathematics* (1896) and *History of Mathematics* (1894), and the English translation of Fink's *Brief History of Mathematics* (1900) are also valuable works for those who read English only. The student of history of mathematics is compelled to examine journals devoted to such history, if he wants to be up to date. The best of these journals at the present time is the *Bibliotheca Mathematica*.¹

The Teaching of Mathematics.—The most striking feature of the current mathematical literature is that such a large amount of it is devoted to teaching. This is doubtless largely due to the influence of the work which is being done under the direction of the International Commission on the Teaching of Mathematics. This commission was appointed in 1908 and it was able to present 156 of its publications at the recent international mathematical congress. The reports prepared by the twelve American committees have been published by the United States Bureau of Education and have been distributed freely. They constitute a valuable addition to the literature which should be read by all teachers of mathematics.

¹ An example of valuable historical material appearing first in a journal is Professor Cajori's "*History of the Logarithmic and Exponential Concepts*" now being published by the MONTHLY, and not yet completed when this address was delivered.

Notwithstanding the great importance of the recent pedagogical literature we cannot pursue this subject further, since our aim has been to confine ourselves practically to mathematics as a branch of knowledge. In closing we may call attention to a type of literature which is often helpful to awaken interest where other means fail. I refer to the puzzles and curiosities which are apparently isolated from any important theory. A large collection of these is found, for instance, in the fifth edition (1911) of Ball's *Mathematical Recreations and Essays*.

BOOK REVIEWS.

The Teaching of Mathematics in Secondary Schools. By ARTHUR SCHULTZE.

The Macmillan Company, New York, 1912. xx+370 pages. \$1.25.

Instead of treating the general problems which face a teacher of mathematics in the high school, the author has chosen to provide a text which covers in detail the teaching of algebra and geometry. This makes the book of particular value to the inexperienced teacher.

With this end in view, the theoretical discussion has been cut down to the mere essentials. The present inefficiency of mathematical teaching, the reasons for retaining mathematics in our high school course, the different methods of teaching with their advantages and difficulties, the logical foundations of algebra and geometry, are rapidly reviewed. Such a brief summary must, of necessity, consist of little more than a statement of facts. An outline of this kind might be rendered more valuable both to old and new teachers by references to more detailed discussion of certain topics. Such references the author has in most cases failed to provide.

For the inexperienced teacher, the first book of plane geometry is full of difficulties. The author has attempted to smooth out some of these by a very detailed treatment. He discusses the characteristics of a good definition, and points out the difficulty in forming a definition. He also gives a number of examples to illustrate the preliminary definitions. The author would not have too great stress laid on the first few propositions. He would rather assume their proof and then impress their value by means of simple exercises.

The author insists that the original exercise is the important part of geometrical teaching, for it is only through it that the student gains power to think, and more than this, the original is more under the control of the teacher than the ordinary proposition. Methods of forming exercises are given and a large number of graded problems on the first propositions in triangles are added.

The discussion of the propositions of the first book centers about the equality of triangles and the properties of parallel lines. The treatment includes not only methods of proof but such incidental matters as converses of propositions, use of symbols and logical arrangement. Throughout the discussion in book one, the author uses the arrangement of propositions of Schultze and Sevenoak's *Geometry*. This renders the work less valuable to those teachers who use a text with radically different arrangement.